3.4

(a)

\[ \frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2E}{dk^2} \]

\[ \frac{1}{m^*} \sim \text{curvature} \]

curvature of A < curvature of B

So, \( m_A^* > m_B^* \)

(b)

3.7

(a) Larger effective mass by smaller curvature.

See the graph below. Green dashed line is \( ak^2 \) and red solid line is \( ak^2 - bk^4 \).

(b) Smaller curvature \( \rightarrow \) larger effective mass.

The effective mass of GaAs increases at energies only slightly removed from Ec as described in Table 3.1 footnote.
4.3

(a) No change for those equations, but $k_z$ is discretized due to the small size of z-dimension.

(b)

Density of states of 2D quantum well is discontinuous at $E_{2D}$ and

$$E_{2D} = E_{3D} + E_n = E_{3D} + \left( \frac{\hbar^2}{2m} \right) \left( \frac{n\pi}{c} \right)^2.$$

(c) – ii

Density of states

$$0 \ (0 \leq E \leq E_1), \quad \frac{m}{2\pi\hbar^2} (E_1 \leq E \leq E_2), \quad \frac{m}{2\pi\hbar^2} (E_n \leq E \leq E_{n+1})$$

(d)
4.14

(a) \( T = 300K, \ N_A \ll N_D, \ N_D = 10^{14} / cm^3 \)

\[
n = 10^{14} / cm^3
\]

\[
p = \frac{n^2}{n} = \frac{(10^{10})^2}{10^{14}} = 10^6 / cm^3
\]

(b) \( T = 300K, \ N_A = 10^{15} / cm^3, \ N_D \ll N_A \)

\[
p = 10^{15} / cm^3
\]

\[
n = \frac{n^2}{p} = \frac{(10^{10})^2}{10^{15}} = 10^5 / cm^3
\]

(c) \( T = 300K, \ N_A = 9 \times 10^{15} / cm^3, \ N_D = 10^{16} / cm^3 \)

\[
n = \left[ \left( \frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2} + \frac{N_D - N_A}{2} = 10^{15} / cm^3
\]

\[
p = \left[ \left( \frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2} - \frac{N_D - N_A}{2} = 10^5 / cm^3
\]

(d) \( T = 470K, \ N_A = 0, \ N_D = 10^{14} / cm^3 \)

\[\text{from Table 4.2}\]

\[N_c = 2 \left( \frac{2\pi m^* kT}{\hbar^2} \right)^{3/2} \rightarrow N_{c,470K} = N_{c,300K} \left( \frac{470}{300} \right)^{3/2} = 3.23 \times 10^9 \times \left( \frac{470}{300} \right)^{3/2} = 6.33 \times 10^{10} / cm^3\]

\[N_v = 2 \left( \frac{2\pi m^* kT}{\hbar^2} \right)^{3/2} \rightarrow N_{v,470K} = N_{v,300K} \left( \frac{470}{300} \right)^{3/2} = 1.83 \times 10^9 \times \left( \frac{470}{300} \right)^{3/2} = 3.59 \times 10^{10} / cm^3\]

\[n_i = \sqrt{N_c N_v} \exp \left( - \frac{E_g}{2kT} \right) = \sqrt{(6.33 \times 10^{10})(3.59 \times 10^{10})} \exp \left( - \frac{1.12}{2 \times 0.026 \times (470/300)} \right) = 5.10 \times 10^{13} / cm^3\]

\[
n = \left[ \left( \frac{N_D}{2} \right)^2 + n_i^2 \right]^{1/2} + \frac{N_D}{2} = 1.21 \times 10^{14} / cm^3
\]

\[
p = \frac{n^2}{n} = 2.15 \times 10^{13} / cm^3
\]

(e) \( T = 645K, \ N_A = 0, \ N_D = 10^{14} / cm^3 \)
\[ N_{c,645K} = N_{c,300K} \left( \frac{645}{300} \right)^{3/2} = 3.23 \times 10^{19} \times \left( \frac{645}{300} \right)^{3/2} = 1.02 \times 10^{20} / \text{cm}^3 \]

\[ N_{v,645K} = N_{v,300K} \left( \frac{645}{300} \right)^{3/2} = 1.83 \times 10^{19} \times \left( \frac{645}{300} \right)^{3/2} = 5.77 \times 10^{19} / \text{cm}^3 \]

\[ n_i = \sqrt{N_c N_v} \exp\left( -\frac{E_g}{2 kT} \right) = \sqrt{(1.02 \times 10^{20})(5.77 \times 10^{19})} \exp\left( -\frac{1.12}{2 \times 0.026 \times (645/300)} \right) = 3.42 \times 10^{15} / \text{cm}^3 \]

\[ n_i \gg N_A, N_D \]

\[ n = p = n_i = 3.42 \times 10^{15} / \text{cm}^3 \]

4.20
(a)

\[ N_v = 2 \left( \frac{m_p k_B T}{2 \pi \hbar^2} \right)^{3/2}, \quad N_c = 2 \left( \frac{m_n k_B T}{2 \pi \hbar^2} \right)^{3/2} \]

\[ E_i = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln \left( \frac{N_v}{N_c} \right) = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln \left( \frac{m_p}{m_n} \right)^{3/2} \]

\[ m_p > m_n \rightarrow E_i \text{ is closer to } E_c. \]

(b)

\[ E_i = E_m + \frac{k_B T}{2} \ln \left( \frac{m_p}{m_n}^* \right)^{3/2} \rightarrow E_i - E_m = \frac{k_B T}{2} \ln \left( \frac{m_p}{m_n}^* \right)^{3/2} = 0.069 \text{ eV} \]

\[ \therefore E_i \text{ is 0.069 eV above midgap.} \]

(c)

\[ E_c \quad E_i \quad 0.069 \text{ eV} \quad E_m \quad 0.18 \text{ eV} \quad E_v \]
We assumed a nondegenerate semiconductor, but $E_i$ is in degenerate region. \((E_c - E_i < 3k_B T)\).

(d)

$$N_v = 2 \left( \frac{2 \pi m^*_p k_B T}{\hbar^2} \right)^{3/2} = 6.33 \times 10^{18} \text{ /cm}^3$$

From a view of holes, it’s nondegenerate.

$$p = N_v \exp \left( \frac{E_v - E_F}{k_B T} \right)$$

$$n_i = N_v \exp \left( \frac{E_v - E_i}{k_B T} \right) \rightarrow E_i - E_v = -k_B T \ln \left( \frac{n_i}{N_v} \right) = 0.155 \text{ eV}$$

$E_i$ is 0.155 eV above $E_v$ or 0.065 eV above $E_m$.

(e)

$$N_D \ll n_i \rightarrow E_F = E_i$$

$\therefore E_F$ is 0.065 eV above $E_m$. 